

# Chapter 10

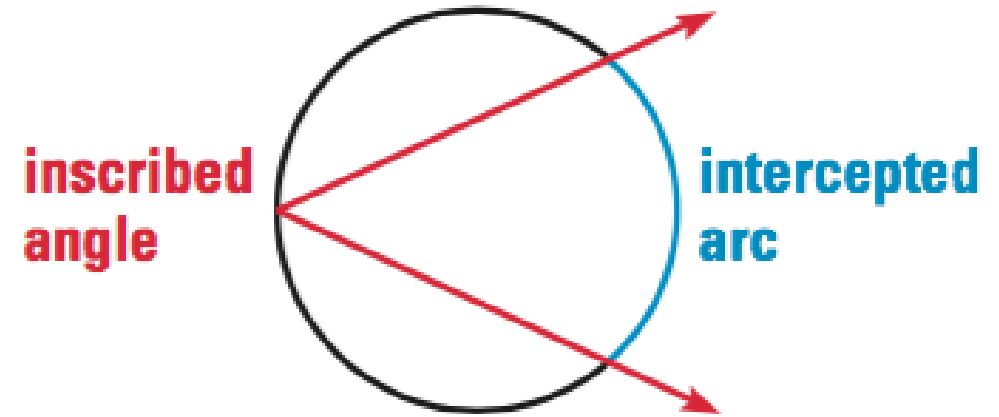
## Circles

# Section 3

## Inscribed Angles

## GOAL 1: Using Inscribed Angles

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



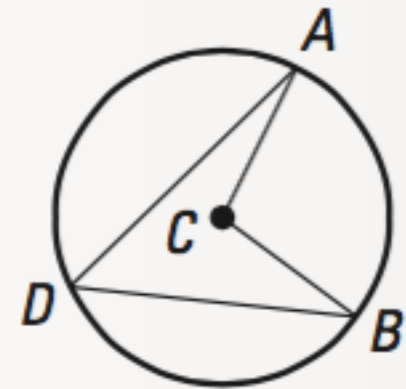
### THEOREM

#### THEOREM 10.8 *Measure of an Inscribed Angle*

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

$$m\angle ADB = \frac{1}{2}m\widehat{AB}$$

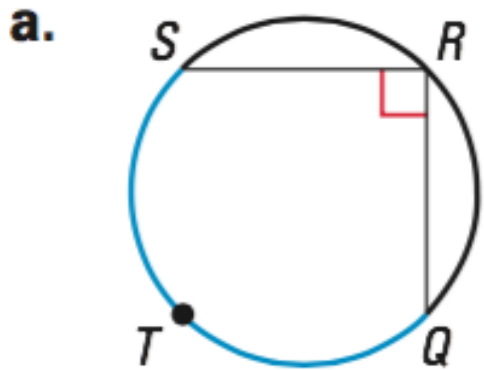
$$\angle = \frac{1}{2} \text{ arc} \quad (2\angle = \text{arc})$$



## Example 1: Finding Measures of Arcs and Inscribed Angles

\*given angle  $\rightarrow$   $\times$  by 2 to find arc; given arc  $\rightarrow$   $/$  by 2 to find angle\*

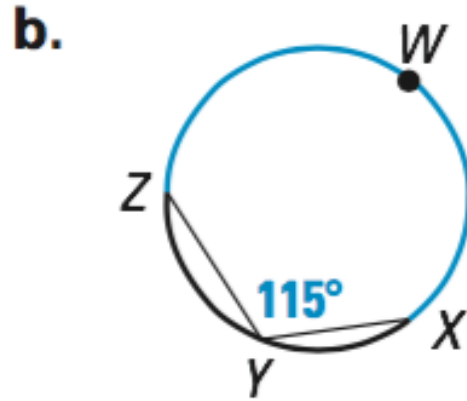
Find the measure of the blue arc or angle.



$$\angle = \frac{1}{2} \cap$$

$$90 = \frac{1}{2} \cap$$

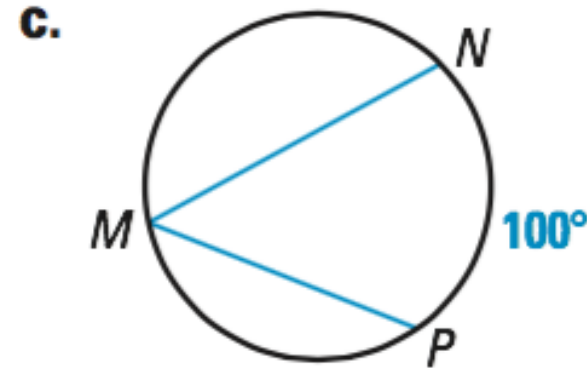
$$\widehat{STQ} = 180^\circ$$



$$\angle = \frac{1}{2} \cap$$

$$115 = \frac{1}{2} \cap$$

$$\widehat{ZW} = 230^\circ$$



$$\angle = \frac{1}{2} \cap$$

$$\angle = \frac{1}{2} (100)$$

$$\angle M = 50^\circ$$

## Example 2: Comparing Measures of Inscribed Angles

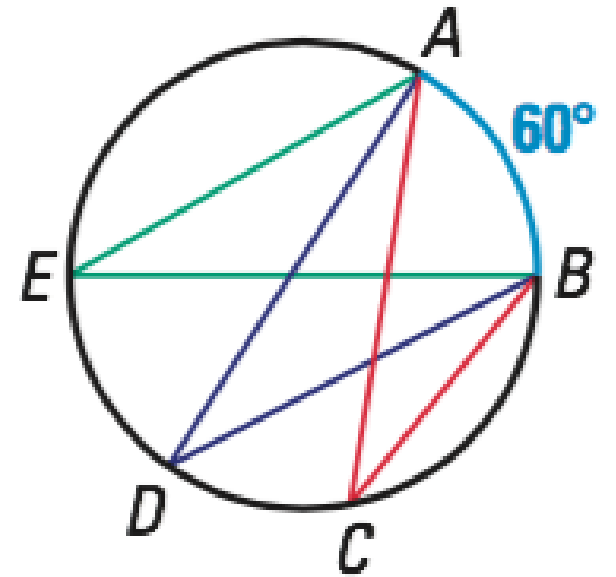
Find  $m\angle ACB$ ,  $m\angle ADB$ , and  $m\angle AEB$ .

$$\angle = \frac{1}{2} \text{ arc}$$

$$\angle = \frac{1}{2} (60)$$

$$\angle = 30^\circ$$

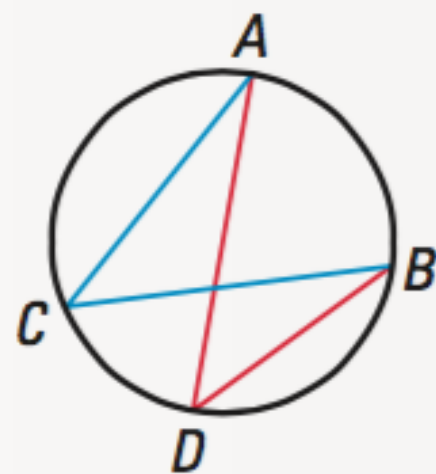
$$\angle ACB = \angle ADB = \angle AEB = 30^\circ$$



## THEOREM

### THEOREM 10.9

If two **inscribed** angles of a circle intercept the same arc, then the angles are congruent.



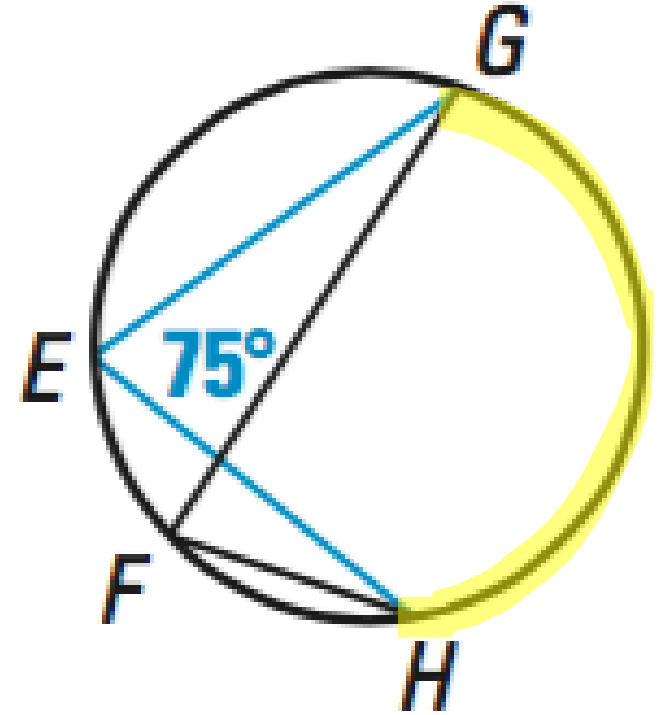
$$\angle C \cong \angle D$$

### Example 3: Finding the Measure of an Angle

It is given that  $m\angle E = 75^\circ$ . What is  $m\angle F$ ?

$$m\angle F = 75^\circ$$

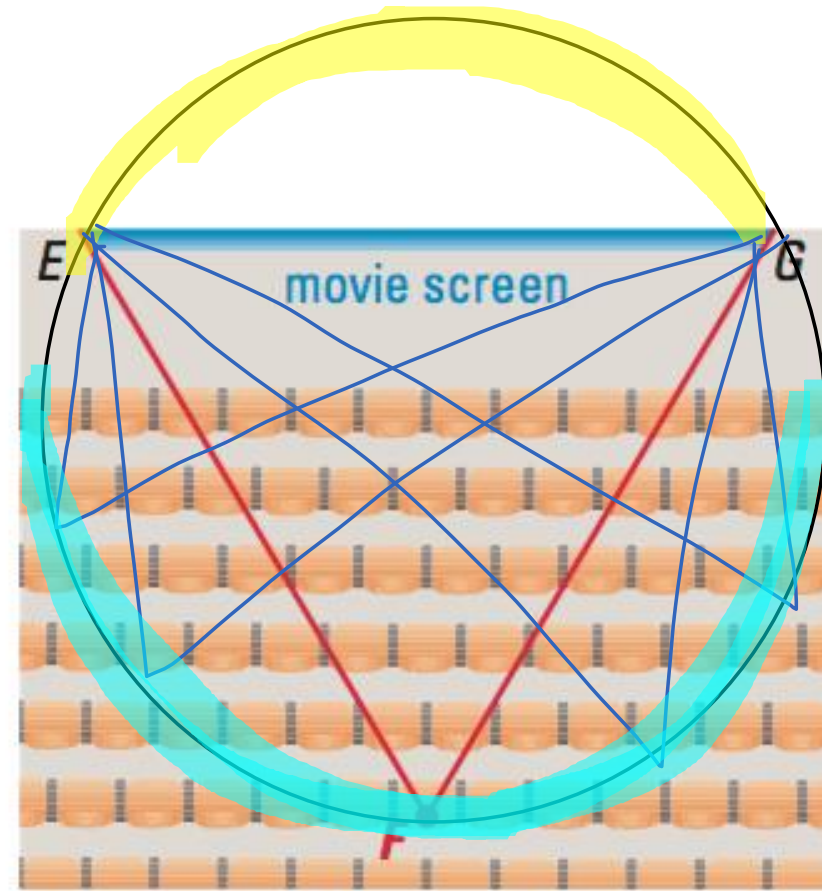
(they both intercept  $\widehat{GH}$ )



## Example 4: Using the Measure of an Inscribed Angle

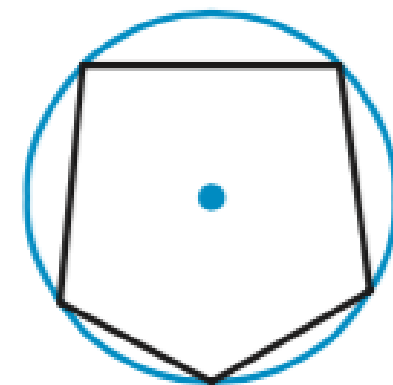
**THEATER DESIGN** When you go to the movies, you want to be close to the movie screen, but you don't want to have to move your eyes too much to see the edges of the picture. If  $E$  and  $G$  are the ends of the screen and you are at  $F$ ,  $m\angle EFG$  is called your *viewing angle*.

You decide that the middle of the sixth row has the best viewing angle. If someone is sitting there, **where else can you sit to have the same viewing angle?**



## GOAL 2: Using Properties of Inscribed Polygons

If all of the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an *inscribed polygon* and the circle is a *circumscribed circle*. You are asked to justify Theorem 10.10 and part of Theorem 10.11 in Exercises 39 and 40. A complete proof of Theorem 10.11 appears on page 840.

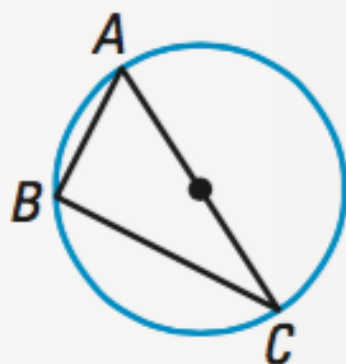


## THEOREMS ABOUT INSCRIBED POLYGONS

### THEOREM 10.10

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

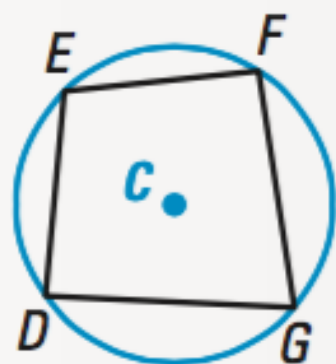
$\angle B$  is a right angle if and only if  $\overline{AC}$  is a diameter of the circle.



### THEOREM 10.11

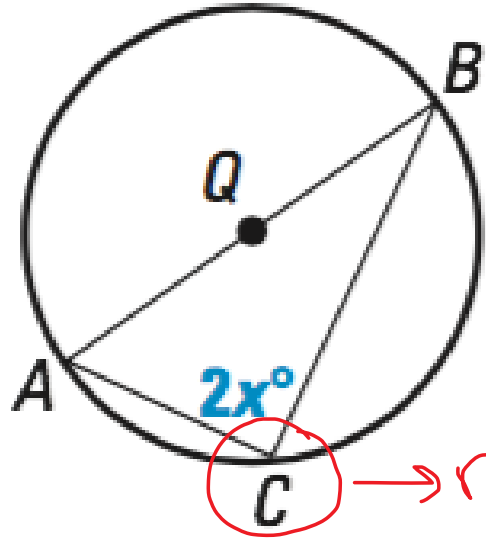
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

$D, E, F,$  and  $G$  lie on some circle,  $\odot C$ , if and only if  $m\angle D + m\angle F = 180^\circ$  and  $m\angle E + m\angle G = 180^\circ$ .



Example 5: Using Theorems 10.10 and 10.1  
Find the value of each variable.

a.

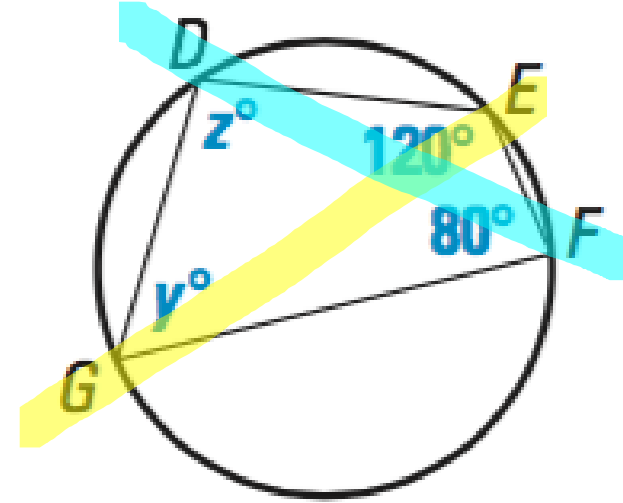


$\rightarrow$  right  $\angle$

$$\frac{2x}{2} = \frac{90}{2}$$

$$x = 45$$

b.



$$G: 180 - 120 = 60^\circ$$

$$D: 180 - 80 = 100^\circ$$

## Example 6: Using an Inscribed Quadrilateral

In the diagram, ABCD is inscribed in Circle P. Find the measure of each angle.

$$\begin{aligned} 3(5x + 2y) &= 180 \\ 2(3x + 3y) &= 180 \end{aligned}$$

$$\begin{array}{r} 15x + 6y = 540 \\ - (6x + 6y = 360) \\ \hline 9x = 180 \\ \underline{9} \quad \quad \underline{9} \end{array}$$

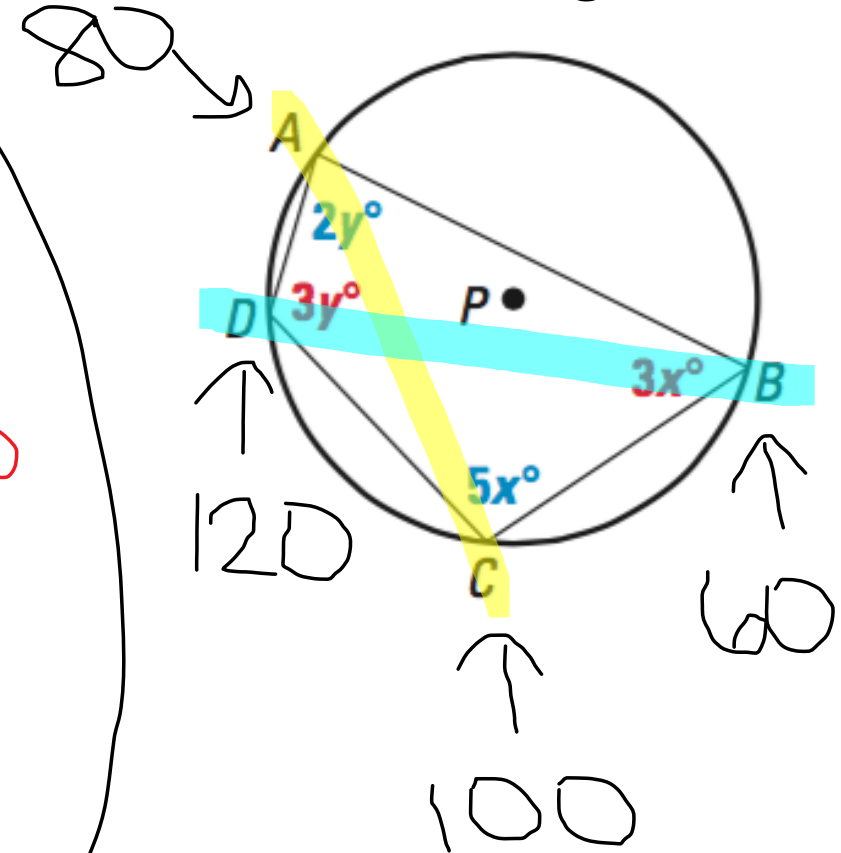
$$x = 20$$

$$5(20) + 2y = 180$$

$$\begin{array}{r} 100 + 2y = 180 \\ -100 \quad \quad -100 \\ \hline 2y = 80 \end{array}$$

$$\frac{2y}{2} = \frac{80}{2}$$

$$y = 40$$



EXIT SLIP